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Heavy quark potential and quarkonia dissociation rates

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Abstract. Quenched lattice data for the $Q\bar{Q}$ interaction (in terms of heavy quark free energies) in the color-singlet channel at finite temperatures are fitted and used within the non-relativistic Schrödinger equation formalism to obtain binding energies and scattering phase shifts for the lowest eigenstates in the charmonium and bottomonium systems in a hot gluon plasma. The partial dissociation rate due to the Bhanot–Peskin process is calculated using different assumptions for the gluon distribution function, including free massless gluons, massive gluons, and massive damped gluons. It is demonstrated that a temperature dependent gluon mass has an essential influence on the heavy quarkonia dissociation, but that this process alone is insufficient to describe the heavy quarkonia dissociation rates.

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1 Introduction

Heavy quarkonia have been suggested as hard probes of the quark–gluon plasma [1] since the modification of static interactions at finite temperature eventually implies a dissolution of heavy quarkonia bound states into the continuum of scattering states (Mott effect).

The dissolution of quarkonium bound states in heavy-ion collisions results in an observable suppression of heavy quarkonium production. Since the Mott temperatures for J/ψ, Υ and Υ' as obtained by solving the Schrödinger equation for a screened Cornell-type potential lie well above the critical temperature T_{c} for deconfinement [2], it had soon been realized that a kinetic theory is necessary for the description of heavy quarkonia dissociation [3]; see [4,5] for recent formulations. Solutions of the Schrödinger equation provide the basis for the evaluation of cross sections and rates for the Bhanot–Peskin process [6] of heavy quarkonia dissociation by gluon impact [7]. In this contribution we present a new fit to the singlet free energies from quenched lattice QCD simulations [8,9] and show that binding energies and cross sections deviate from those obtained for Debye potential fits [10–12]. Recently spectral function analyses of quarkonia within lattice QCD have revealed that the J/ψ survives as a well identifiable resonance above its Mott temperature [13,14]. We present results of a phase shift analysis which suggest the presence of correlations even above the Mott temperature and are thus in qualitative agreement with this observation. Alternative attempts to solve the apparent discrepancy while neglecting scattering state contributions to the resonance spectral function argued for changing the potential in the Schrödinger equation in order to obtain higher Mott temperatures; see the review by Karsch [15].

2 Heavy quark potential

The main source of our knowledge of the static interaction for a heavy quark–antiquark ($Q\bar{Q}$) system at high temperature are calculations of the Polyakov-loop correlator from lattice QCD.

The color-singlet part F_1 of the free energy of a quark–antiquark system is obtained from the equation

$$\langle \mathrm{Tr}[L(0)L^{\dagger}(r)]\rangle = \exp[-F_1(r)/T] \qquad (1)$$

and is interpreted hereafter as the effective static $Q\bar{Q}$ interaction potential in a hot gluon plasma.

2.1 Zero temperature

At zero temperature, we will consider the color-singlet heavy quark potential $V_1(r)$ in the Schrödinger equation in order to describe the heavy quarkonia bound state spectrum. By fitting the spectrum the two unknown parameters can be determined: the heavy quark mass and a constant shift of the whole potential. The behavior of the quark–antiquark interaction in the color-singlet channel was investigated in [16] within the combined lattice and

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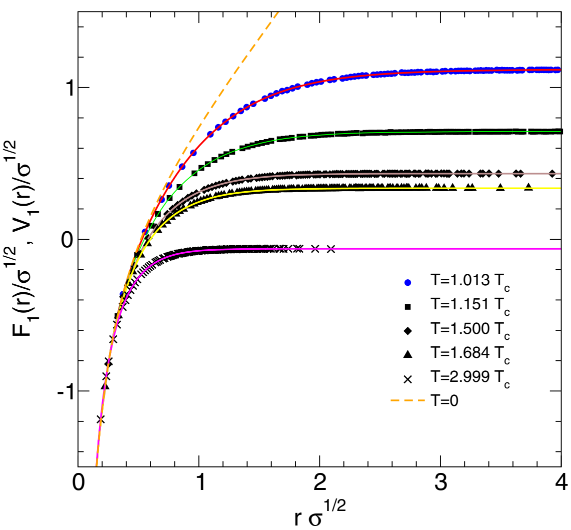

Fig. 1. Lattice data for the singlet free energy [9] at different temperatures; see legend. Solid lines are obtained with the fit on the basis of the Dixit formula (7), the dashed line is the $T = 0$ potential $V_1(r)$

perturbative approach in quenched QCD. We fit these data points implementing a χ^2 minimization to the Ansatz

$$V_1(r) = \begin{cases} V_{1,\mathrm{short}}(r), \; r < r_0, \\ V_{1,\mathrm{long}}(r), \;\; r \geq r_0, \end{cases} \tag{2}$$

where $V_{1,\mathrm{short}}(r)$ describes the interaction at short distances whereas $V_{1,\mathrm{long}}(r)$ is responsible for the long-distance forces. Both expressions are matched at $r = r_0$ which is defined below. This point, as we shall see, lies in the domain of perturbative QCD. We use the combined linear and Coulomb potential to describe the long-distance interaction and the Coulomb interaction with the r-dependent coupling constant $\alpha(r)$ for short distances:

$$V_{1,\mathrm{long}}(r) = \sigma r - \frac{\pi}{12 r}, \tag{3}$$

$$V_{1,\mathrm{short}}(r) = -\frac{4}{3}\frac{\alpha(r)}{r}, \tag{4}$$

$$\alpha(r) = \frac{4\pi}{11}\left(\frac{1}{\ln\left(r^2/c^2\right)} - \frac{r^2}{r^2 - c^2}\right). \tag{5}$$

The formula for $\alpha(r)$ is obtained by solving the one-loop renormalization group equation for the running coupling constant in QCD followed by the pole subtraction [17]. The constant $c\sqrt{\sigma} \approx 1.816$ and the point $r_0\sqrt{\sigma} \approx 0.031$ in units of the string tension $\sqrt{\sigma} = 0.42\,\mathrm{GeV}$ are determined from the condition that the potential is a smooth function at $r = r_0 \approx 0.0146\,\mathrm{fm}$. The result is given by the dashed line in Fig. 1.

2.2 Singlet free energy at high temperature

For the singlet free energy of a static quark–antiquark system at high temperature ($T > T_{\mathrm{c}}$) we assume that the short-range interaction governed by pQCD coincides with the zero-temperature form of the previous subsection whereas the long-distance interaction $F_{1,\mathrm{long}}(r,T)$ requires theoretical assumptions about its shape. Instead of the frequently used screened Coulomb potential, we follow Dixit [18] and assume the potential behavior at large r as follows:

$$F_{1,\mathrm{long}}(r,T) = -\frac{q^2(T)}{2^{3/4}\Gamma(3/4)}\sqrt{\frac{r}{\mu(T)}}K_{1/4}\left[(\mu(T)r)^2\right] + q^2(T)\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)\mu(T)}, \tag{6}$$

where $K_{1/4}(x)$ is a modified Bessel function. A similar parameterization was used by Digal et al. [19]. We add to this term a screened Coulomb attraction at short distances and obtain

$$F_1(r,T) = F_{1,\mathrm{long}}(r,T) + V_{1,\mathrm{short}}(r)\mathrm{e}^{-(\mu(T)r)^2}. \tag{7}$$

The two temperature dependent parameters $q(T)$ and $\mu(T)$ are determined from a fit to the lattice data [9]; their behavior for $1.5 \leq T/T_{\mathrm{c}} \leq 3.0$ can be represented as $\mu(T)/\sqrt{\sigma} = 0.540 + 0.778T/T_{\mathrm{c}}$, $q^2(T)/\sigma = 1.57 - 0.592T/T_{\mathrm{c}}$; $T_{\mathrm{c}} = 0.264\,\mathrm{GeV}$. In Fig. 1 we compare lattice data with the fit for $F_{1,\mathrm{long}}(r,T)$ at selected temperatures.

3 Heavy quarkonia

3.1 Quarkonia at zero temperature

The masses of quarkonia in the vacuum are defined as

$$M = 2m_Q + E + v_0, \tag{8}$$

where m_Q is the quark mass, and the energy E is an eigenvalue of the Schrödinger equation

$$[-\nabla^2/m_Q + V(r)]\psi(r) = E\psi(r), \tag{9}$$

where $V(r)$ is identified with the zero-temperature potential $V_1(r)$ of (2) up to an unknown constant v_0. Substituting the wave function $\psi_{n\ell m}(r,\theta,\phi) = r^{-1}R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$ into (9), one obtains an equation for $R_{n\ell}(r)$. At large r, the potential is linear, and the solution of this equation behaves as the Airy function $R_{n\ell}(r) \sim \mathrm{Ai}(\kappa r - \xi)$, where $\kappa^3 = m_Q\sigma$ and $\xi = m_Q E/\kappa^2$.

The masses of $1S$ and $4S$ states [20] are used as input. For charmonium we obtain $m_c = 1.45\,\mathrm{GeV}$ and the constant $v_0 = -0.302\,\mathrm{GeV}$. For bottomonium we have $m_b = 4.785\,\mathrm{GeV}$ with the same v_0. Once m_c, m_b, and v_0 are fixed, the remaining quarkonia spectrum is well described [21]. Note that in the purely gluonic system (quenched QCD) at $T = 0$, there is no continuum threshold from which the binding energy E_{B} could be defined.

3.2 Quarkonia at finite temperature

The Schrödinger equation for a bound state in the QGP has the form

$$[-\nabla^2/m_Q + V_{\mathrm{eff}}(r,T)]\psi(r,T) = E_{\mathrm{B}}(T)\psi(r,T), \tag{10}$$

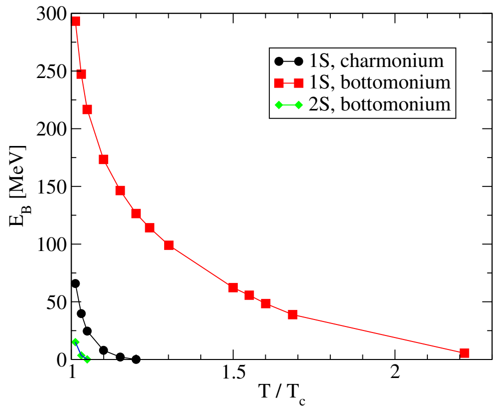

Fig. 2. Binding energies of heavy quarkonia states from solution of the Schrödinger equation for the T-dependent effective potential of Fig. 1

where $E_{\mathrm{B}}(T) > 0$ is the temperature dependent binding energy. The medium effects on the $Q\bar{Q}$ system are modeled using the singlet free energies [9] as an effective potential $V_{\mathrm{eff}}(r,T) = F_1(r,T) - F_\infty(T)$ with the continuum threshold $F_\infty(T) = \lim_{r\to\infty} F_1(r,T)$. The temperature dependent mass of a quarkonium bound state is defined as

$$M(T) = 2m_Q - E_{\mathrm{B}}(T) + v_0 + F_\infty(T). \qquad (11)$$

The solutions for the binding energy both for charmonium and bottomonium are shown in Fig. 2.

The wave function of an unbound quark–antiquark system can be calculated via the S-wave phase shift function $\delta_S(r)$ by solving the equation [22]

$$\frac{\mathrm{d}\delta_S(k,r,T)}{\mathrm{d}r} = -\frac{m_Q V_{\mathrm{eff}}(r,T)}{k}\left[\sin(kr + \delta_S(k,r,T))\right]. \qquad (12)$$

The phase shift is defined as $\delta_S(k,T) \equiv \delta_S(k,\infty,T)$ and results are shown in Fig. 3 for charmonia and bottomonia states at different temperatures. In accordance with the Levinson theorem, the scattering phase shift at threshold changes by π once a bound state merges the continuum at its Mott temperature; $T^{\mathrm{Mott}}/T_{\mathrm{c}} \approx 1.05, 1.20, 2.25$ for Υ', J/ψ, Υ, respectively; see Figs. 2 and 3.

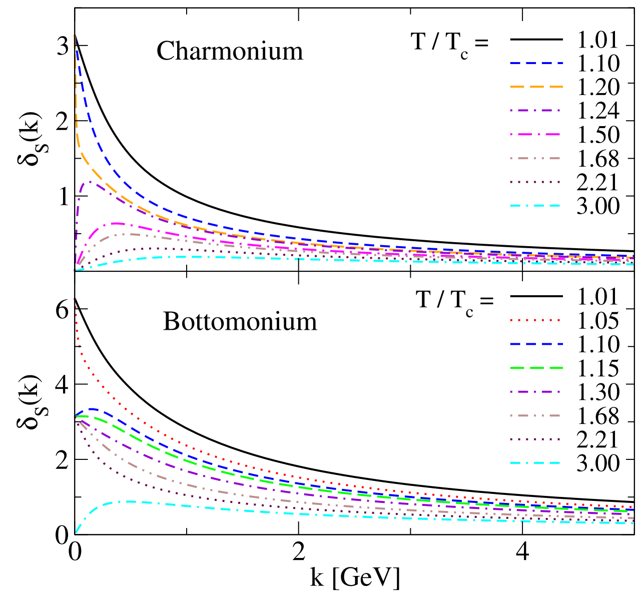

Fig. 3. Scattering phase shifts for S-wave charmonia and bottomonia states from solution of the Schrödinger equation for the T-dependent potential of Fig. 1. A jump of $\delta_S(0)$ by π signals the dissolution of a bound state into the continuum

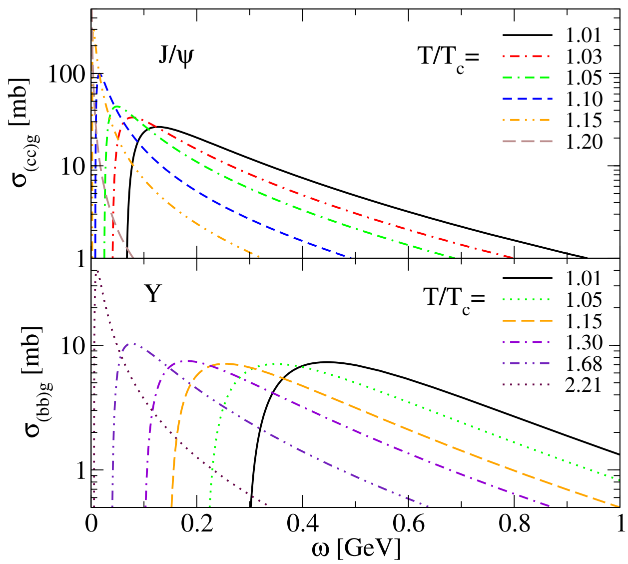

Fig. 4. Cross sections for J/ψ and Υ dissociation by the Bhanot–Peskin process

4 Dissociation of quarkonia by gluon impact

We calculate the cross section for the Bhanot–Peskin process [6] similarly to the case of the deuteron photodissociation [11,23]

$$\sigma_{(Q\bar{Q})g}(\omega) = \frac{4\pi\alpha_{gQ}}{3}\frac{(k^2+k_0^2)}{k}\left(\int_0^\infty u_{1P}(r)u_{1S}(r)\,r\,\mathrm{d}r\right)^2, \qquad (13)$$

$$\int_0^\infty |u_{1S}(r)|^2\mathrm{d}r = 1,\ \alpha_{gQ} = \alpha_{\mathrm{s}}/6,\ k_0^2 = m_Q E_{\mathrm{B}}(T), \qquad (14)$$

where we used $R_{n\ell}(r) = u_{n\ell}(r)\mathrm{e}^{-k_0 r}$. For the $1P$ state, one can use the wave function of a free $Q\bar{Q}$ system:

$$u_{1P}(r) = \frac{\sin kr}{kr} - \cos kr, \quad k^2 = m_Q(\omega - E_{\mathrm{B}}(T)). \qquad (15)$$

For the constant α_{s} in (14), we take an average over the low energy region below 1 GeV, which gives $\alpha_{\mathrm{s}} \approx 0.48$. As a result, we obtain the cross sections shown in Fig. 4. Their peak values correspond to the geometrical ones ($\pi R^2(T)$) with the T-dependent radius $R(T)$ of the quarkonia wave function [21].

Now we can estimate the dissociation rate of the charmonium and bottomonium by gluon impact according to

$$\Gamma_{(\bar{Q}Q)g}(T) = \int_0^\infty \mathrm{d}s A(s) \int_s^\infty \frac{\mathrm{d}\omega^2}{4\pi^2}\sqrt{\omega^2 - s}\,\sigma_{(Q\bar{Q})g}(\omega) n_g(\omega), \qquad (16)$$

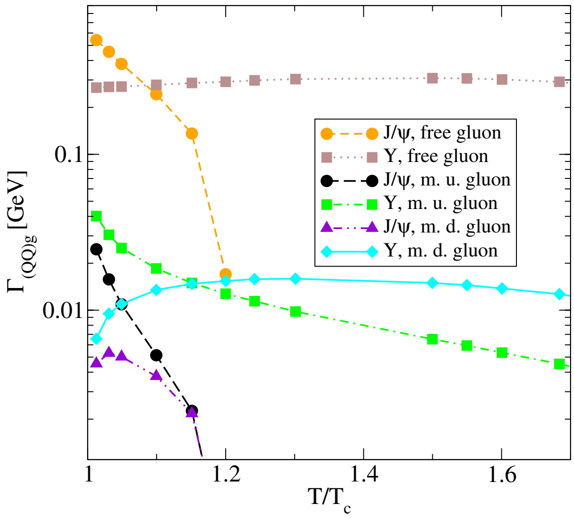

Fig. 5. Dissociation rates of heavy quarkonia in a hot gluon plasma obtained with massive damped (m.d.), massive undamped (m.u.) and massless (free) gluon distributions in (16), respectively

where $A(s)$ is the normalized gluon spectral function

$$A(s) = \frac{1}{\pi} \frac{\sqrt{s}\gamma}{(s - m_g^2)^2 + s\gamma^2}, \qquad \int_0^\infty \mathrm{d}s A(s) = 1, \qquad (17)$$

and the thermal gluon distribution function is given by $n_g(\omega) = 2(N_c^2 - 1)[\exp(\omega/T) - 1]^{-1}$. The temperature dependent gluon mass m_g and damping width γ are taken from a recent fit to lattice QCD data for the entropy in pure gauge [24], $m_g^2 = 2\pi\bar{\alpha}T^2$, $\gamma = 3\bar{\alpha}T\ln(2.67/\bar{\alpha})$, where $\bar{\alpha} = 12\pi/[33\ln(3.7(T/T_c - 0.67))^2]$. The results are shown in Fig. 5 and compared with the cases when the damping width and also the gluon mass are neglected.

5 Conclusions

We have used a new fit to recent quenched lattice data for the $Q\bar{Q}$ singlet free energies at finite temperatures to obtain binding energies and scattering phase shifts for the lowest eigenstates in the charmonium and bottomonium systems within the Schrödinger equation formalism. In contrast to results on the basis of a Debye potential fit, we obtain much smaller finite temperature quarkonia binding energies, entailing large dissociation cross sections for the Bhanot–Peskin process. The corresponding dissociation rates have been evaluated using different assumptions for the gluon distribution function, including free massless gluons, massive gluons, and massive damped gluons. We have demonstrated that a temperature dependent gluon mass has an essential influence on the heavy quarkonia dissociation. However, the Bhanot–Peskin process alone is insufficient to describe the quarkonium dissociation process [5,7]. On the basis of the spectrum and wave functions obtained here we will study next the $Q\bar{Q}$ spectral functions above the Mott temperature and compare the results with corresponding lattice studies [13–15].

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